

# Dynamical Relaxation of the Cosmological Constant and Matter Creation in the Universe

Robert Brandenberger<sup>1,2,3</sup> and Anupam Mazumdar<sup>3</sup>

<sup>1</sup> *Department of Physics, Brown University, Providence, Rhode Island 02912, USA*

<sup>2</sup> *Perimeter Institute, 35 King St. North, Waterloo, ON, N2J 2W9, Canada,*

<sup>3</sup> *Department of Physics, McGill University,  
3600 University Road, Montréal, QC, H3A 2T8, Canada*

In this Letter we discuss the issues of the graceful exit from inflation and of matter creation in the context of a recent scenario [9] in which the back-reaction of long wavelength cosmological perturbations induces a negative contribution to the cosmological constant and leads to a dynamical relaxation of the bare cosmological constant. The initially large cosmological constant gives rise to primordial inflation, during which cosmological perturbations are stretched beyond the Hubble radius. The cumulative effect of the long wavelength fluctuations back-reacts on the background geometry in a form which corresponds to the addition of a negative effective cosmological constant to the energy-momentum tensor. In the absence of an effective scalar field driving inflation, whose decay can reheat the Universe, the challenge is to find a mechanism which produces matter at the end of the relaxation process. In this Letter, we point out that the decay of a condensate representing the order parameter for a “flat” direction in the field theory moduli space can naturally provide a matter generation mechanism. The order parameter is displaced from its vacuum value by thermal or quantum fluctuations, it is frozen until the Hubble constant drops to a sufficiently low value, and then begins to oscillate about its ground state. During the period of oscillation it can decay into Standard Model particles similar to how the inflaton decays in scalar-field-driven models of inflation.

BROWN-HET-1396

## I. INTRODUCTION

The cosmological constant problem remains one of the most important problems of theoretical physics. The problem is to find an explanation for the fact that the value of the cosmological constant in the present Universe - based on observational upper bounds on the contribution of the cosmological constant to the “energy” content of the present Universe - is many orders of magnitude smaller than what is predicted according to theoretical estimates for the vacuum energy (which acts like a cosmological constant). In non-supersymmetric theories, the mismatch is by a factor of  $10^{-120}$  (see e.g. [1,2] for reviews). Recent data, both from supernovae observations [3] and also from cosmic microwave anisotropy measurements (see [4] for the most recent observational results) indicate that the Universe is right now entering a new stage of acceleration, indicating the presence of something that acts as an effective cosmological constant with the value  $\Lambda_{eff} \sim 10^{-120} M_p^4$ . Thus, there now appear to be two aspects of the cosmological constant problem, firstly why the cosmological constant is so tiny compared to theoretical expectations (the “old” cosmological constant problem - using Weinberg’s language [5]) and the coincidence problem of why the remnant cosmological constant is becoming visible precisely at the present time of cosmic history (the “new” cosmological constant problem) and why it does not exactly vanish.

Independent studies of the back-reaction effects of long-wavelength gravitational waves [6] and of long-

wavelength cosmological perturbations [7] on the background geometry of space-time have led to a scenario [8,9] in which these fluctuations lead to a dynamical relaxation of an initially large bare cosmological constant. The large initial cosmological constant leads to a period of primordial inflation. During this period, quantum vacuum fluctuations of both gravitons and scalar metric fluctuations are stretched beyond the Hubble radius, thus generating a large phase space of long-wavelength modes (see e.g. [10,11] for reviews of the theory of cosmological fluctuations). It can be shown that these modes have a back-reaction effect on the local geometry which is analogous to that of a negative cosmological constant (this effect is not physically measurable in models with only one matter field [12–14], but it is physically measurable in models with two or more matter fields [15]). This opens up the possibility that back-reaction may lead to a dynamical relaxation of the bare cosmological constant. What has been shown so far is only the perturbative onset of the relaxation process (see e.g. [16] for a modeling of non-perturbative effects), but if the relaxation mechanism holds beyond perturbation theory, it will lead [9] to the inevitable prediction of a dynamical scaling solution

$$\Omega_{\Lambda_{eff}}(t) \sim 1, \quad (1)$$

(where  $\Omega_X = \rho_X/\rho_c$  is a measure of the contribution of  $X$  to the closure density  $\rho_c$  of the Universe,  $\rho_X$  denoting the effective energy density in some  $X$  “matter”) for the effective cosmological constant, valid at all sufficiently late times  $t$ . Thus, this dynamical relaxation mechanism

would automatically address both the old and the new cosmological constant problems.

The reason why we obtain the dynamical fixed point solution (1) is as follows: as the phase of inflation proceeds, the phase space of infrared modes builds up. Since long-wavelength fluctuations are frozen, the contribution of an individual mode to the effective cosmological constant does not decrease in absolute magnitude. Thus, the total back-reaction effect builds up gradually, and the effective cosmological constant, which is the sum of the positive bare cosmological constant and the induced negative back-reaction term, grows. However, the sum cannot drop to zero, since before this happens the energy density  $\rho_\Lambda$  corresponding to the *effective* cosmological constant will drop below the matter energy density. As soon as this happens, inflation will end, the phase space of infrared modes will cease to increase, and the back-reaction contribution to  $|\Lambda_{eff}|$  levels off. Since the Universe is still expanding, the matter energy density  $\rho_m$  (where “matter” here stands for both cold matter and radiation) continues to decrease, thus allowing  $\rho_\Lambda$  to start dominating again, enabling the back-reaction effect to again increase in strength. Thus, as explained in detail in [9], we expect that at all sufficiently late times  $\rho_\Lambda/\rho_m$  will be oscillating in time about the value 0.5. This is a “tracking” behavior very similar to what is postulated in tracking quintessence models [17] \*.

However, since during the period of inflation the energy density in cold matter and radiation decreases exponentially, there needs to be a mechanism to provide a “graceful exit” from inflation and to reheat the Universe. Otherwise, the matter energy density during the late time “tracking” period will be much too low (note that in order for back-reaction of infrared modes to have an important effect, the period of inflation has to be very long - see [7] for the precise numbers).

In this Letter we propose a solution to this problem which is completely natural from the point of view of the Minimal Supersymmetric Standard Model (MSSM) and many other models of particle physics beyond the Standard Model. These models contain light scalar field (massless in the absence of supersymmetry breaking). During the period of inflation, these fields will form condensates which are displaced from the absolute minima of the respective potentials (after supersymmetry breaking). The displacements may be due to the pre-inflationary initial conditions, or they may develop during inflation via quantum fluctuations. During inflation the dynamical evolution of a condensate is effectively

frozen, but after inflation its dynamics will become important. Specifically, at a value of the Hubble expansion rate  $H$  determined by the potential of the condensate (see Section 3 for details) - much smaller than the initial Hubble expansion rate set by the value of the bare cosmological constant - the energy density of the condensate starts to dominate over the energy density associated with the effective cosmological constant, and the inflationary period will end. At a comparable time, the condensate will become “unfrozen” and begin to oscillate about its ground state. The decay of the condensate into radiation will reheat the Universe by a mechanism very similar to what happens in the final decay of the inflaton condensate in scalar-field-driven inflation.

Particle physics provides ready examples of how to implement our ideas. For example, in the MSSM,  $F$ - and  $D$ -flat directions can form a condensate during inflation (for cosmological consequences of condensates, see [20]). Since the condensate can carry MSSM charges, its decay will automatically give rise to a relativistic bath of Standard Model degrees of freedom. This decay should reheat the Universe to temperature higher than that when the synthesis of light nuclei become important [21]. Note that, in this context, our mechanism provides a natural explanation for the reheating of the Universe into a thermal bath with the right degrees of freedom required for big bang nucleosynthesis (BBN).

We mention two more ways to realize our scenario. The first is by making use of light Majorana neutrinos. In this case, the supersymmetric partner of right handed Majorana neutrino, the sneutrino, can act as a condensate, provided the lightest of the sneutrinos is lighter than the Hubble expansion rate during inflation [22,23]. The second utilizes the moduli fields (scalar fields which are massless before supersymmetry breaking) which arise in the low energy limit of string theories (see e.g. [24]) in which the extra spatial dimensions are compactified.

The outline of this paper is as follows. We first review the mechanism by which cosmological fluctuations back-react on the expansion rate of the Universe. In Section 3 we discuss the dynamics of the condensate and its decay. In Section 4 we conclude with a discussion of some of the challenges for late time cosmology in our scenario.

## II. REVIEW OF THE BACK-REACTION FORMALISM

It has been known for a long time [25] that gravitational waves carry energy and momentum and can have a back-reaction effect on the background in which they propagate. In a similar way, we expect cosmological fluctuations (scalar metric fluctuations) to be described by an energy-momentum tensor which influences the evolution of the background.

The formalism to discuss this back-reaction in the case of cosmological perturbations was initially developed in

---

\*Note that a similar scenario where the effective cosmological constant fluctuates and whose associated energy density always tracks that of matter emerges from the causal set approach to quantum gravity [18,19]. Whereas in our approach the effective cosmological constant must remain positive, in the causal set approach its sign fluctuates.

[7]. The idea is to consider small fluctuations of the metric and the matter fields about a homogeneous and isotropic background, to insert these fluctuations into the Einstein equations and expand these equations to second order in the small parameter  $\epsilon$  which parameterizes the amplitude of the fluctuations. Due to their non-linearity, the Einstein equations are not satisfied at second order, and it is necessary to add back-reaction terms which are quadratic in  $\epsilon$ . In particular, one needs to add a second order correction to the zero mode of the metric. The sum of the background metric plus the quadratic zero mode correction defines a new homogeneous metric  $g_{\mu\nu}^{(0,br)}$  which takes into account the effects of back-reaction to this order. This new metric obeys the modified equations

$$G_{\mu\nu}(g_{\alpha\beta}^{(0,br)}) = 8\pi G \left[ T_{\mu\nu}^{(0)} + \left\langle T_{\mu\nu}^{(2)} - \frac{1}{8\pi G} G_{\mu\nu}^{(2)} \right\rangle \right], \quad (2)$$

where  $G_{\mu\nu}$  indicates the Einstein tensor,  $T_{\mu\nu}$  is the energy-momentum tensor of matter, the pointed brackets stand for spatial averaging, and the superscripts indicate the order in perturbation theory. The terms inside the pointed brackets form what is called the effective energy-momentum tensor  $\tau_{\mu\nu}$  of cosmological perturbations<sup>†</sup> Note that all Fourier modes of the gravitational potential  $\Phi$  contribute to the effective energy-momentum tensor for back-reaction: the effect of fluctuations is cumulative. Thus, back-reaction can be important even if the relative magnitude of each metric fluctuation mode is small (as observations indicate) as long as there is a sufficiently large phase space of infrared modes, i.e. as long as the period of primordial inflation is sufficiently long.

For long wavelength fluctuations, the terms involving the metric fluctuations are dominant. It was shown in [7] that the back-reaction equation is covariant under linear space-time coordinate transformations. Thus, to simplify the analysis, we can work in a convenient gauge, namely Longitudinal gauge in which the metric is diagonal

$$ds^2 = (1 + 2\Phi)dt^2 - a(t)^2(1 - 2\Phi)\delta_{ij}dx^i dx^j, \quad (3)$$

where  $\Phi(x, t)$  is the metric perturbation and  $a(t)$  is the scale factor.

In addition to the metric fluctuations, there are matter fluctuations. One matter fluctuation degree of freedom (the “adiabatic” matter perturbation) is related to the metric potential  $\Phi$  via the Einstein constraint equation. For example, in the case of a matter model with a single

scalar field  $\varphi$ , the matter field fluctuation  $\delta\varphi$  is related to  $\Phi$  via

$$\dot{\Phi} + H\Phi = 4\pi G\dot{\varphi}_0 \delta\varphi. \quad (4)$$

Making use of the background equation of motion for the inflaton field  $\varphi$  during the slow rolling phase, one can replace  $\dot{\varphi}$  in (4) by the scalar field potential  $V(\varphi)$  and its derivative. Since on large scales  $\dot{\Phi} \simeq 0$ , the fluctuation variables  $\delta\varphi$  and  $\Phi$  are directly related, and all terms in the effective energy-momentum tensor  $\tau_{\mu\nu}$  can be expressed as functions of the background variables multiplying  $\langle \Phi^2 \rangle$ .

In hindsight, it is easy to understand why the back-reaction of infrared modes of cosmological perturbations acts as a negative cosmological constant. For long wavelength fluctuations, all terms in the effective energy-momentum tensor involving spatial derivatives are negligible. Since on long wavelengths the amplitude of the metric fluctuation variable  $\Phi$  is frozen (see e.g. [10]), all terms involving temporal derivatives of the fluctuation variables also vanish. The only terms which survive are gravitational potential energy terms. Since matter fluctuations produce negative potential wells, and since for infrared modes the negative gravitational energy dominates over the positive matter energy, the total effective energy density is negative, and the associated energy-momentum tensor takes the equation of state of a potential energy term, i.e. that of a cosmological constant.

In the long-wavelength limit for fluctuations, and the slow-roll approximation for the dynamics of the matter field  $\varphi$  (with potential energy function  $V(\varphi)$ ), the expression for the effective energy density  $\rho_{eff}$  becomes [7]

$$\rho_{br} \equiv \tau_0^0 \simeq \left( 2 \frac{V''V^2}{V'^2} - 4V \right) \langle \Phi^2 \rangle, \quad (5)$$

which is typically negative for shallow potentials. The effective energy-momentum tensor  $\tau_{\mu\nu}$  takes the form of a *negative cosmological constant*

$$p_{br} = -\rho_{br} \text{ with } \rho_{br} < 0. \quad (6)$$

A crucial observation is that the magnitude of  $\rho_{br}$  increases as a function of time. This is because, in an inflationary Universe, as time increases more and more wavelengths become longer than the Hubble radius and begin to contribute to  $\rho_{br}$ .

However, as first pointed out in [12] (see also [26]) and later shown rigorously in [13] and [14], in single matter field models the back-reaction of the dominant infrared terms is not physically measurable. It can be locally masked by a time translation. This can be seen by studying the back-reaction of long wavelength cosmological perturbations on a local measure of the expansion rate, and expressing the result in terms of a local clock. However, if there is an additional matter field present which can be used to define a physical clock (this

---

<sup>†</sup>Note that even if inflation is driven by the bare cosmological constant, we need to introduce a matter field in order to have scalar metric fluctuations. For simplicity, we shall introduce a scalar field  $\varphi$  which is slowly rolling during the period of inflation (but which has a negligible effect on the background dynamics, at least initially).

field may be viewed as describing the temperature of the cosmic microwave background), then the leading back-reaction terms described here are real, as recently shown in [15] (see also [27] for another model which demonstrates that infrared back-reaction is “for real”). Since our model involves at least one additional matter field (namely the condensate), the infrared back-reaction will be physically measurable.

Since it is the total metric and not the background metric only which determines observables, it was suggested [9] on the basis of the above results that the effective cosmological constant at time  $t$  is given by

$$\Lambda_{eff}(t) = \Lambda_0 + \rho_{br}, \quad (7)$$

where the second term on the right hand side is negative and has an absolute value which is increasing in time. Since mode by mode the magnitude of the square of the fluctuation amplitude which enters the expectation value  $\langle \Phi^2 \rangle$  is tiny, the back-reaction contribution to the effective cosmological constant is very small compared to the bare cosmological constant  $\Lambda_0$  during the early stages of inflation. However, if inflation lasts long enough (as will be the case in many inflationary models in the class of “chaotic” inflation [28]), then, before the homogeneous scalar field  $\varphi$  has ended the slow-rolling phase, the back-reaction contribution to the effective cosmological constant will cancel the initial bare value  $\Lambda_0$ . However, as described in the Introduction, the energy density associated with  $\Lambda_{eff}(t)$  can in fact never become negative, and at late times one will obtain a dynamical scaling solution in which the energy density associate with  $\Lambda_{eff}(t)$  tracks the matter energy density.

### III. CONDENSATE DYNAMICS AND REHEATING

As mentioned in the Introduction, in order that the back-reaction scenario for dynamically relaxing the bare cosmological constant produce a Universe compatible with the one we observe today, there needs to be some component of matter whose energy density does not red-shift significantly during primordial inflation. Condensates which are frozen during inflation can naturally play this role. As mentioned in the Introduction, the existence of such condensate (or moduli) fields is a generic feature of many particle physics theories beyond the Standard Model.

In this paper we assume a generic flat direction field,  $\chi$ , which is lifted by a small mass  $m_\chi$ . We assume that the mass is small compared to the Hubble expansion during the early stages of inflation. Therefore  $\chi$  is free to fluctuate along this flat direction. Because inflation smoothes out all gradients, only the homogeneous condensate mode

survives<sup>‡</sup> The fluctuations of the condensate do play a role, however, in that they produce the entropy fluctuations which are crucial [29,15] in order that the back-reaction effect be physically measurable.

While the energy density of the condensate is much smaller than the energy in the bare cosmological constant, the Hubble expansion rate  $H(t)$  is not influenced by  $\chi$ . If  $H(t)$  were constant, then the condensate  $\chi$  would be slowly rolling provided  $m_\chi \ll H$ , i.e. the acceleration term in the equation of motion for  $\chi$

$$\ddot{\chi} + 3H(t)\dot{\chi} + m_\chi^2\chi = 0 \quad (8)$$

would be negligible. However, in our case, as the back-reaction effect grows, the time-dependence of  $H(t)$  will become important and the slow-rolling approximation has to be modified even while  $m_\chi \ll H$ . We make the *modified slow rolling ansatz*

$$\dot{\chi} = -\frac{\alpha m_\chi^2}{3H(t)}\chi, \quad (9)$$

where  $\alpha$  is a constant to be determined ( $\alpha = 1$  if  $H(t)$  is independent of  $t$ ). With this ansatz, the second derivative of  $\chi$  now contains two terms

$$\ddot{\chi} = \left( \frac{m_\chi^2}{3H(t)} \right)^2 \chi + \frac{m_\chi^2}{3H(t)^2} \dot{H}\chi. \quad (10)$$

While  $m_\chi \ll H$ , the first term is negligible. Using the background Friedmann-Robertson-Walker equations to substitute for  $\dot{H}$ , we then find

$$\alpha = \frac{1}{1 + \frac{1+w}{2}}, \quad (11)$$

where  $w(t)$  labels the effective equation of state of the background (in homogeneous isotropic cosmology,  $w = p/\rho$ , where  $p$  is the pressure and  $\rho$  is the energy density).

Integrating Equation (9) from some given time  $t_0$  to time  $t$  then yields the following solution for  $\chi(t)$ :

$$\chi(t) = \chi(t_0) \exp \left( -\alpha m_\chi^2 \int_{t_0}^t H^{-1} dt \right). \quad (12)$$

For a fixed value of  $w$ , the time integral in the exponent for  $t \gg t_0$  yields the value  $\beta/2H(t)^2$ , where

$$\beta = \frac{2}{3(1+w)}, \quad (13)$$

---

<sup>‡</sup>Fluctuations are continuously generated in the ultraviolet, but if the Hubble constant is decreasing, the spectrum of fluctuations will be blue, and the amplitude of the short wavelength fluctuations will rapidly become tiny compared to the size of the “zero mode”. Thus, we will focus on the mode of  $\chi$  which is quasi-homogeneous.

from which it follows that  $\chi(t)$  is effectively frozen until  $H(t) = m_\chi$ .

Once  $H(t)$  falls below  $m_\chi$ , the condensate  $\chi$  will start oscillating about  $\chi = 0$  in a manner analogous to how the inflaton starts oscillating about the minimum of its potential at the end of inflation. If the initial value of the condensate is smaller than the Planck mass  $m_{pl}$ , the energy density during the initial period of oscillation of  $\chi$  will still be dominated by the effective cosmological constant (since  $\rho_\chi < \rho_\Lambda$ ). However, given initial conditions for  $\chi$  as in the chaotic inflation scenario [28], one would expect  $\chi(t_0) \geq m_{pl}$  and thus the energy density in  $\chi$  would begin to dominate before or when  $\chi$  starts oscillating, and the Hubble expansion rate would be determined by the condensate dynamics, which mimics a time-averaged equation of state of dust.

Let us imagine (in analogy with early studies of reheating in inflation [30,31]) that the condensate decays perturbatively into light fermions with a decay width  $\Gamma \sim g^2 m_\chi$ . Then, the energy density in  $\chi$  will be transferred effectively to matter only when

$$\Gamma \sim H. \quad (14)$$

The reheating temperature  $T_{reh}$  (the temperature of ordinary matter after the energy transfer is completed) can then be estimated via

$$T_{reh}^4 \sim \frac{3}{8\pi G} H_{dec}^2, \quad (15)$$

where  $H_{dec}$  is the value of  $H$  when the energy transfer takes place. This yields

$$T_{reh} \sim \left( \frac{3}{8\pi} \right)^{1/4} g m_{pl}^{1/2} m_\chi^{1/2}, \quad (16)$$

where  $m_{pl}$  is the Planck mass. Note that for sufficiently small values of  $m_\chi \ll 10^7$  GeV and for  $g \leq 10^{-4}$ , it is possible to obtain a reheat temperature below  $10^9$  GeV (similar constraints were obtained in [23,32]) which would render our scenario safe from the potential problem of overproduction of thermal and non-thermal gravitinos [33].

On the other hand, for a condensate whose potential is dominated by the mass term (as we assume here), it is likely that the initial stages of the energy transfer from the condensate to matter will proceed via the parametric resonance instability [34]. This occurs both if  $\chi$  is coupled to bosons [35] or to fermions [36]. In this case, the energy density of matter after energy transfer corresponds to a matter temperature  $T_{max}$  of

$$T_{max} \sim m_\chi^{1/2} m_{pl}^{1/2}. \quad (17)$$

In this case, the upper bound on  $m_\chi$  to be safe from the gravitino over-abundance problem is lower than if the decay occurs perturbatively.

In both cases, the decay of the condensate naturally provides us a hot thermal Universe required for BBN.

If the condensate carries some local or global quantum number, such as baryonic charge, then it is possible that the decay yields a baryon to entropy ratio of  $10^{-10}$  which is required for successful BBN (see e.g. [22,23]).

#### IV. CONCLUSION

In this Letter we discussed a natural way of gracefully exiting the period of primordial inflation and of reheating the Universe in the scenario in which the back-reaction of long-wavelength cosmological fluctuations dynamically relaxes a bare cosmological constant which provides a period of primordial inflation. The mechanism makes use of the dynamics of a condensate field  $\chi$  which has a small mass  $m_\chi$  and which has a quasi-homogeneous mode which is excited during the initial stage of inflation. This condensate is frozen until the effective Hubble parameter drops below the condensate mass.

The main role of the condensate is to provide a form of matter energy which is not red-shifted exponentially during the period of primordial inflation. When  $H(t)$  drops below  $m_\chi$ , the energy density of the Universe becomes dominated by the condensate field, thus terminating the period of primordial inflation naturally. At that time, the condensate  $\chi$  begins to oscillate about  $\chi = 0$ , and its decay will reheat the Universe to a temperature determined by  $m_\chi$  and by dimensionless coupling constants which describe the coupling of  $\chi$  to regular matter.

Candidates for the condensate are moduli fields of string compactifications, MSSM flat directions or sneutrino condensates. Such condensates are naturally excited in the context of inflationary cosmology. An added bonus of our scenario is that the decay of the condensate, which starts once the condensate unfreezes, automatically provides a hot thermal bath of Standard Model particles, a prerequisite for successful big bang nucleosynthesis. We show that it is possible to reheat the Universe to below  $10^9$  GeV if the Yukawa coupling has a value  $g \sim 10^{-4}$  and if the effective mass of the condensate satisfies  $\sim 10^7$  GeV. For smaller masses, the reheat temperature can be further lowered, and therefore there is no danger of over-producing thermal/non-thermal gravitinos.

Our work is one step forwards towards connecting the scenario of [9] with a realistic late-time cosmology. However, a lot more work needs to be done in this area. In particular, it is important to study the effects of an effective cosmological constant which is oscillating about  $\Omega_\Lambda = 1/2$  (the dynamical fixed point solution described in Section 2) on big bang nucleosynthesis, structure formation, and the evolution of cosmic microwave background anisotropies <sup>§</sup>. Work on these problems is in

---

<sup>§</sup>Any model with an oscillating effective cosmological con-

progress [37].

### Acknowledgments

R.B. is supported in part by the US Department of Energy under Contract DE-FG02-91ER40688, TASK A. He thanks the Fields Institute for Mathematical Sciences and the Perimeter Institute for hospitality and support during the final stages of this project. A.M. is a CITA National Fellow. His work is also supported in part by NSERC (Canada) and by the Fonds de Recherche sur la Nature et les Technologies du Québec.

- 
- [1] S. Weinberg, *Rev. Mod. Phys.* **61**, 1 (1989).
  - [2] S. M. Carroll, W. H. Press and E. L. Turner, *Ann. Rev. Astron. Astrophys.* **30**, 499 (1992).
  - [3] S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], *Astrophys. J.* **517**, 565 (1999) [arXiv:astro-ph/9812133];  
A. G. Riess *et al.* [Supernova Search Team Collaboration], *Astron. J.* **116**, 1009 (1998) [arXiv:astro-ph/9805201];  
A. G. Riess *et al.* [Supernova Search Team Collaboration], [arXiv:astro-ph/0402512].
  - [4] D. N. Spergel *et al.*, *Astrophys. J. Suppl.* **148**, 175 (2003) [arXiv:astro-ph/0302209].
  - [5] S. Weinberg, arXiv:astro-ph/0005265.
  - [6] N. C. Tsamis and R. P. Woodard, *Annals Phys.* **238**, 1 (1995).
  - [7] V. F. Mukhanov, L. R. W. Abramo and R. H. Brandenberger, *Phys. Rev. Lett.* **78**, 1624 (1997) [arXiv:gr-qc/9609026];  
L. R. W. Abramo, R. H. Brandenberger and V. F. Mukhanov, *Phys. Rev. D* **56**, 3248 (1997) [arXiv:gr-qc/9704037].
  - [8] N. C. Tsamis and R. P. Woodard, *Phys. Lett. B* **301**, 351 (1993);  
N. C. Tsamis and R. P. Woodard, *Nucl. Phys. B* **474**, 235 (1996) [arXiv:hep-ph/9602315].
  - [9] R. H. Brandenberger, Plenary talk at 18th IAP Colloquium on the Nature of Dark Energy: Observational and Theoretical Results on the Accelerating Universe, Paris, France, 1-5 Jul 2002. arXiv:hep-th/0210165;  
R. H. Brandenberger, “Back reaction of cosmological perturbations,” arXiv:hep-th/0004016;  
R. H. Brandenberger, Brown preprint BROWN-HET-1180, contribution to the 19th Texas Symposium on Relativistic Astrophysics, Paris, France, Dec. 14-18, 1998.
  - [10] V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, *Phys. Rept.* **215**, 203 (1992).
  - [11] R. H. Brandenberger, arXiv:hep-th/0306071.
  - [12] W. Unruh, “Cosmological long wavelength perturbations,” arXiv:astro-ph/9802323.
  - [13] L. R. Abramo and R. P. Woodard, *Phys. Rev. D* **65**, 063515 (2002) [arXiv:astro-ph/0109272].
  - [14] G. Geshnizjani and R. Brandenberger, *Phys. Rev. D* **66**, 123507 (2002) [arXiv:gr-qc/0204074].
  - [15] G. Geshnizjani and R. Brandenberger, arXiv:hep-th/0310265.
  - [16] N. C. Tsamis and R. P. Woodard, *Annals Phys.* **267**, 145 (1998) [arXiv:hep-ph/9712331].
  - [17] P. J. Steinhardt, L. M. Wang and I. Zlatev, *Phys. Rev. D* **59**, 123504 (1999) [arXiv:astro-ph/9812313].
  - [18] R. D. Sorkin, *Int. J. Theor. Phys.* **36**, 2759 (1997) [arXiv:gr-qc/9706002].
  - [19] M. Ahmed, S. Dodelson, P. B. Greene and R. Sorkin, arXiv:astro-ph/0209274.
  - [20] K. Enqvist and A. Mazumdar, *Phys. Rept.* **380**, 99 (2003) [arXiv:hep-ph/0209244];  
M. Dine and A. Kusenko, arXiv:hep-ph/0303065.
  - [21] K. Enqvist, S. Kasuya and A. Mazumdar, *Phys. Rev. Lett.* **90**, 091302 (2003);  
K. Enqvist, A. Jokinen, S. Kasuya and A. Mazumdar, *Phys. Rev. D* **68**, 103507 (2003).
  - [22] A. Mazumdar, arXiv:hep-ph/0306026;  
A. Mazumdar, *Phys. Lett. B* **580**, 7 (2004) [arXiv:hep-ph/0308020].
  - [23] A. Mazumdar and A. Perez-Lorezana, arXiv:hep-ph/0311106.
  - [24] J. Polchinski, *String Theory, Vol. 2*, (Cambridge Univ. Press, Cambridge, 1998).
  - [25] D. Brill and J. Hartle, *Phys. Rev.* **135**, B271 (1964);  
R. Isaacson, *Phys. Rev.* **166**, 1272 (1968).
  - [26] N. Afshordi and R. H. Brandenberger, *Phys. Rev. D* **63**, 123505 (2001) [arXiv:gr-qc/0011075].
  - [27] L. R. Abramo and R. P. Woodard, *Phys. Rev. D* **65**, 063516 (2002) [arXiv:astro-ph/0109273].
  - [28] A. D. Linde, *Phys. Lett. B* **129**, 177 (1983).
  - [29] F. Finelli and R. H. Brandenberger, *Phys. Rev. D* **62**, 083502 (2000) [arXiv:hep-ph/0003172].
  - [30] A. D. Dolgov and A. D. Linde, *Phys. Lett. B* **116**, 329 (1982).
  - [31] L. F. Abbott, E. Farhi and M. B. Wise, *Phys. Lett. B* **117**, 29 (1982).
  - [32] K. Enqvist, S. Kasuya and A. Mazumdar, arXiv:hep-ph/0311224.
  - [33] J. R. Ellis, J. E. Kim and D. V. Nanopoulos, *Phys. Lett. B* **145**, 181 (1984);  
A. L. Maroto and A. Mazumdar, *Phys. Rev. Lett.* **84**, 1655 (2000) [arXiv:hep-ph/9904206];  
R. Kallosh, L. Kofman, A. D. Linde and A. Van Proeyen, *Phys. Rev. D* **61**, 103503 (2000) [arXiv:hep-th/9907124].
  - [34] J. Traschen, R. Brandenberger, *Phys. Rev. D* **42**, 2491 (1990).
  - [35] L. Kofman, A. D. Linde and A. A. Starobinsky, *Phys. Rev. Lett.* **73**, 3195 (1994) [arXiv:hep-th/9405187];
- 

stant, such as the model of [18], will have to address these issues.

- Y. Shtanov, Ukr. Fiz. Zh. **38**, 1425 (1993) (in Russian);  
Y. Shtanov, J. Traschen, and R. Brandenberger, Phys.  
Rev. D **51**, 5438 (1995);  
L. Kofman, A. D. Linde and A. A. Starobinsky, Phys.  
Rev. D **56**, 3258 (1997) [arXiv:hep-ph/9704452].  
[36] P. B. Greene and L. Kofman, Phys. Lett. B **448**, 6 (1999)  
[arXiv:hep-ph/9807339].  
[37] A. Mazumdar and R. Brandenberger, work in progress.